1) A population of protozoa develops with a constant relative growth rate of 0.7944 per member per day. On day zero the population consists of two members. Find the population size after six days.

$$\boxed{\text{About } 235}$$

2) A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria.

   a) Find an expression for the number of bacteria after t hours. $\boxed{y(t) = 500(16)^{t/3}}$

   b) Find the number of bacteria after 4 hours. $\boxed{\approx 20{,}159}$

   c) Find the rate of growth after 4 hours. $\boxed{\approx 18{,}631 \text{ cells/h}}$

   d) When will the population reach 30,000? $\boxed{\approx 4.4 \text{ h}}$

3) The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.

    a) Find the mass that remains after t years. $\boxed{y(t)=100(2)^{-t/30}}$

    b) How much of the sample remains after 100 years? $\boxed{\approx 9.92 \text{ mg}}$

    c) After how long will only 1 mg remain? $\boxed{\approx 199.3 \text{ years}}$

4) A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F.

    a) If the temperature of the turkey is 150°F after half an hour, what is the temperature after 45 minutes?

    b) When will the turkey have cooled to 100°F?

$$\boxed{\begin{array}{l} a) \approx 137°\text{F} \\ b) \approx 116 \text{ min} \end{array}}$$

5) Consider a population $P = P(t)$ with constant relative birth and death rates α and β, respectively, and a constant emigration rate m, where α, β, and m are positive constants. Assume that $\alpha > \beta$. Then the rate of change of the population at time t is modeled by the differential equation:

$$\frac{dP}{dt} = kP - m \qquad \text{where } k = \alpha - \beta$$

a) Find the solution of this equation that satisfies the initial condition $P(0) = P_0$.

b) What condition on m will lead to an exponential expansion of the population?

c) What condition on m will result in a constant population? A population decline?

d) In 1847, the population of Ireland was about 8 million and the difference between the relative birth and death rates was 1.6% of the population. Because of the potato famine in the 1840s and 1850s, about 210,000 inhabitants per year emigrated from Ireland. Was the population expanding or declining at that time?

$$
\begin{array}{l}
a)\ \ P(t) = \dfrac{m}{k} + \left(P_0 - \dfrac{m}{k} \right) e^{kt} \\[2ex]
b)\ \ m < kP_0 \\[2ex]
c)\ \ \text{Constant: } m = kP_0,\ \text{Decline: } m > kP_0 \\[2ex]
d)\ \ \text{Declining}
\end{array}
$$